

**Carderock Division**  
**Naval Surface Warfare Center**

Bethesda, Md. 20084-5000

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**NSWCCD-TM-80-96/06 April 1996**

Machinery Research and Development Directorate

Technical Memorandum

**Water Droplet Evaporation in Air**  
**During Compression in a Gas Turbine Engine**

by  
Earl Quandt

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# WATER DROPLET EVAPORATION IN AIR DURING COMPRESSION IN A GAS TURBINE ENGINE

BY: EARL QUANDT

## I. Introduction

A water fog concept is being considered for evaporative cooling of the air as it is compressed in a ship gas turbine engine. The following analysis is presented to clarify the physics associated with liquid droplet evaporation in this situation, to understand the conditions affecting the cooling, and to identify any further information required to achieve such a concept.

The vaporization of small liquid drops in a warm ideal gas is controlled by the outward motion of the vapor and the inward flow of heat to cause evaporation. Following the standard analysis of Spalding, as given in "Principles of Combustion" by Kuo\*, it is assumed that the process is "quasi steady." This means that the conditions far removed from the drop are constant, and that there are no time varying terms in the Eulerian description of the mass and energy flows. For a spherically symmetric drop we have:

$$\begin{array}{ll} \text{Conservation of Mass:} & d(\bar{\rho}vr^2) = 0 \\ \text{(Total)} & \end{array}$$

$$\begin{array}{ll} \text{Conservation of Mass:} & r^2 \bar{\rho}_v \frac{dY_1}{dr} = \frac{d}{dr} \left( r^2 \bar{\rho} D \frac{dY_1}{dr} \right) \\ \text{(Vaporizing Phase)} & \end{array}$$

$$\begin{array}{ll} \text{Conservation of Mass:} & r^2 \bar{\rho}_v \frac{dY_2}{dr} = \frac{d}{dr} \left( r^2 \bar{\rho} D \frac{dY_2}{dr} \right) \\ \text{(Ideal Gas Phase)} & \end{array}$$

$$\begin{array}{ll} \text{Conservation of Energy:} & r^2 \bar{\rho}_v \frac{dC_p T}{dr} = \frac{d}{dr} \left( r^2 \bar{\rho} \alpha \frac{dC_p T}{dr} \right) \\ \text{(Total)} & \end{array}$$

It is assumed throughout that  $p_T \approx \text{constant}$ , and that  $\alpha = \frac{k}{\bar{\rho} C_p} = D = \text{constant}$ ,  $Y_1 + Y_2 = 1$ ,  $Y_1 \ll Y_2$ .

It will also be necessary to assume that at the droplet surface the vapor pressure/temperature relationship follows that of the pure vaporizing component.

## II. Analysis - (Quasi-Steady)

Solution of the above mass and energy equations is possible by specifying boundary conditions at the drop surface and at a large distance from the surface. Then, these solutions can be joined by the vapor pressure - temperature relationship at the surface. This gives the "quasi-

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\*Kuo, Kenneth K., "Principles of Combustion," John Wiley & Sons, 1986.

steady" evaporation rate, and the "lifetime" of a droplet of a given size and mass. The steps in the standard analysis are given by integration of the conservation equations.

$$\bar{\rho}vr^2 = \text{const} = \frac{\dot{m}}{4\pi} = \frac{\dot{m}_1}{4\pi}$$

$$(\bar{\rho}vr^2) Y_1 = \bar{\rho}r^2 D \frac{dY_1}{dr} + C_1$$

$$(\bar{\rho}vr^2) Y_2 = \bar{\rho}r^2 D \frac{dY_2}{dr} + C_2$$

$$(\bar{\rho}vr^2) T = \bar{\rho}r^2 \alpha \frac{dT}{dr} + C_3$$

Here the similarity of the equations for  $Y_1$  and  $T$  is clear, and one only needs to insert suitable boundary conditions and normalizations to obtain identical solutions. It should also be noted that only the vaporizing component contributes to the radial mass flow.

- Mass Concentration

For the water/air system of interest, we have

$$v = \frac{1}{\bar{\rho}} = v_1 Y_1 + v_2 Y_2 = v_2 Y_2 \left( 1 + \frac{v_1 Y_1}{v_2 Y_2} \right)$$

$$\therefore \bar{\rho} = \bar{\rho}_2$$

Thus, for constant pressure, the mean density in the gaseous phase is approximately constant as long as  $Y_1 \ll Y_2$ , and  $T$  does not vary too much.

Solving for the several constants, we introduce the boundary conditions at the drop surface,  $r = R$ ,

$$C_1 = \frac{\dot{m}}{4\pi} \left( Y_{1R} - \frac{D}{v_R} \frac{dY_1}{dr} \right) \Bigg|_{r=R}$$

$$C_2 = \frac{\dot{m}}{4\pi} \left( Y_{2R} - \frac{D}{v_R} \frac{dY_2}{dr} \right) \Bigg|_{r=R}$$

and, since  $Y_1 + Y_2 = 1$ ,

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$$C_2 = \frac{\dot{m}}{4\pi} \left( 1 - Y_{IR} + \frac{D}{v_R} \frac{dY_1}{dr} \Big|_{r=R} \right) = \frac{\dot{m}}{4\pi} - C_1$$

Now, we recognize  $C_2$  as proportional to the mass flux of the ideal gas phase, which is zero, so  $C_1 = \dot{m}/4\pi$ , and

$$- \frac{D}{v_R} \frac{dY_1}{dr} \Big|_{r=R} = (1 - Y_{IR})$$

So, to normalize the terms we let

$$b(r) \equiv \frac{-Y_1(r)}{1 - Y_{IR}}$$

Therefore we have  $db/dr|_{r=R} = v_R/D$ , and

$$(r^2 \bar{\rho} v) b = (r^2 \bar{\rho} v) \frac{D}{v} \frac{db}{dr} - \frac{C_1}{1 - Y_{IR}}$$

So that at  $r = R$ ,

$$(r^2 \bar{\rho} v) (b_R - 1) = - \frac{C_1}{1 - Y_{IR}}$$

and in general

$$(r^2 \bar{\rho} v) (b - b_R + 1) = r^2 \bar{\rho} D \frac{db}{dr}$$

This equation can be solved by separating the variables and integrating from  $r = \infty$  to  $r$ .

$$\int_{\infty}^r \frac{dr}{r^2} = \int_{\infty}^r \frac{D}{(\bar{\rho} r^2 v)} \frac{db}{(b - b_R + 1)}$$

$$- \left( \frac{1}{r} - \frac{1}{r_{\infty}} \right) = \frac{\bar{\rho} D}{(\bar{\rho} r^2 v)} \ln \left( \frac{b - b_R + 1}{b_{\infty} - b_R + 1} \right)$$

Therefore

$$\frac{(b - b_R + 1)}{(b_{\infty} - b_R + 1)} = e^{-rv/D}$$


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and when  $r = R$ ,  $b = b_R$ , and

$$(b_{\infty} - b_R + 1) = e^{Rv_R/D}$$

so that

$$b_R - b_{\infty} = -e^{Rv_R/D} = 1 - e^{\dot{m}/4\pi\rho D}$$

and

$$b_R - b(r) = 1 - e^{\left(\frac{1}{R} - \frac{1}{r}\right) \dot{m}/4\pi\rho D} = \frac{Y_1(r) - Y_{1R}}{1 - Y_{1R}}$$

Thus we have the mass concentration of  $Y_1$  defined in terms of radial location and evaporation rate. It is of interest to note that  $Y_{1R}$  is not 1.0 at the drop surface.

- Temperature Distribution

Turning to the thermal situation, the equation solution is similar, and is related to the boundary conditions at the droplet surface and for large distances from the drop. Since at the surface the heat conduction through the stationary ideal gas phase provides the energy for evaporation, at  $r = R$ ,  $T = T_R$ , and

$$-\lambda \left. \frac{dT}{dr} \right|_{r=R} = -\frac{\dot{Q}}{A} = -h_v \frac{\dot{m}}{A} = -h_v v_R \rho_R$$

Thus at the surface

$$(\bar{\rho} v r^2) T_R = \bar{\rho} R^2 \alpha h_v \bar{\rho} v_R / \lambda + C_3$$

$$C_3 = \frac{\dot{m}}{4\pi} \left( T_R - \frac{h_v}{C_p} \right)$$

Substituting the thermal constant,  $C_3$ , into the temperature differential equation gives

$$\left( T - T_R + \frac{h_v}{C_p} \right) = \frac{\alpha}{v} \frac{dT}{dr} = \frac{\alpha r^2}{v_R R^2} \frac{dT}{dr}$$

Separating variables and integrating from  $r = \infty$  to  $r = r$  gives;

$$\ln \left( \frac{T - T_R + h_v/c_p}{T_{\infty} - T_R + h_v/c_p} \right) = -\frac{v_R R^2}{\alpha} \left( \frac{1}{r} - \frac{1}{r_{\infty}} \right)$$

Thus at  $r = R$ ,  $T = T_R$  and

$$(T_\infty - T_R) h_v / c_p + 1 = e^{v_R R / \alpha}$$

and

$$(T_\infty - T_R) h_v / c_p = e^{\dot{m} / 4\pi R \rho \alpha} - 1 = b_\infty - b_R$$

such that

$$\frac{(T - T_R + h_v / c_p)}{T_\infty - T_R + h_v / c_p} = e^{-\dot{m} / 4\pi \bar{\rho} \alpha r}$$

and

$$\frac{T_{(r)} - T_R}{h_v / c_p} = e^{\frac{\dot{m}}{4\pi \alpha \bar{\rho}} \left( \frac{1}{R} - \frac{1}{r} \right)} - 1 = \frac{Y_{1R} - Y_{1(r)}}{1 - Y_{1R}}$$

Therefore, if  $\alpha = D$ , the temperature and vapor mass fraction solutions are equivalent, if one knows the constant mass evaporation rate,

- Vapor - Liquid Equilibria

Although the equations for temperature and vapor mass fraction give similar functional solutions, we still need an additional relationship between  $T$  and  $Y_1$  to give the absolute values. This can be obtained from the condition that at the droplet surface the mass fraction of the vapor is given by its partial pressure for that temperature.

$$Y_{1R} = \frac{\rho_{1R}}{\rho} = \frac{n_1}{n} \frac{W_1}{W} = \frac{P_{1R}}{P_T} \frac{W_1}{W} = \frac{P_1(T_{1R})}{P_T} \frac{W_1}{W}$$

using the Clausius Clapeyron relationship,

$$\frac{P_{1(T)}}{P_{ref}} = e^{-\frac{h_v}{RT_{ref}}} \left( 1 - \frac{T_{ref}}{T} \right)$$

it is possible to express the vapor mass fraction at the surface in terms of the droplet surface temperature. For a reference condition on the vapor-liquid equilibrium line of 100°C for water, the reference pressure is 1 atm, so

$$h_v \approx 556 \text{ cal/g} \quad R \approx 0.073 \text{ cal/g}^\circ\text{k} \quad T_{Ref} = 373^\circ\text{k}$$

and we let  $T = 373 + \delta t$ , so that

$$P_1 = e^{\frac{556}{.073 \times 373} \left( 1 - \frac{T_{\text{Ref}}}{T_{\text{Ref}} + \delta t} \right)} \approx e^{20.4 \left( \frac{\delta t}{T_{\text{Ref}}} \right)} \text{ atm}$$

Thus,

$$Y_{1R} \approx \frac{W_1}{W} \frac{P_{\text{Ref}}}{P_T} e^{20.4 \left( \frac{T_R}{373} - 1 \right)}$$

so, we now have three equations in  $Y_{1R}$ ,  $T_R$  and  $v_R$  or  $\dot{m}_1$  so there will be a closed solution for  $R$ ,  $P_T$ ,  $T_\infty$ , and the properties.

### III. Application

In order to illustrate the solution procedure we will examine the evaporative cooling behavior for a water spray injected into the compressor. The initial conditions will be air at 100°F and 50% relative humidity. We will look first at the evaporation rate for a single stage with pressure ratio of 1.3.

$$\frac{T_2}{T_0} = (1.3)^{\frac{\gamma-1}{\gamma}} \approx 1.08 \quad T_0 = 311\text{K}, T_2 = 336\text{K} \text{ \& } T = 324\text{K}$$

At 100°F, the vapor pressure of water is 0.95 psi, so that 50% RH corresponds to  $p_v = 0.475 \approx 78^\circ\text{F}$ , and

$$Y_{1\infty} = \frac{0.475}{14.7} \cdot \frac{18}{29} = 0.02 \text{ g/g}_t$$

Adjusting the Clapeyron equation to 100°F = 311K,  $p_{\text{ref}} = 0.95$  psi,  $h_v = 1040$  BTU/LB = 578 cal/g

$$P_{1(t)} = 0.95 e^{25.5 \left( \frac{\delta t}{311} \right)}$$

Now, with pressurization, the water vapor initially in the air is also pressurized to  $\approx 0.63$  psi, or a saturation temperature of 87°F. So, any temperature above 87°F is available to cause evaporation. At the same time, the temperature rise of the isentropic compression has made it possible for the air to hold 0.11 gwater/g, so the new RH is 18%. Therefore  $T$  of 87° is a convenient reference for the vapor pressure, since  $\delta t=0$  results in no concentration potential for  $Y_1$ .



$$P_1 = 0.63 e^{26.4 \frac{\delta t}{304}} \& Y_{1R} = 0.021 e^{26.4 \left( \frac{T_R - 304}{304} \right)}$$

$$\frac{(T_\infty - 304) - (T_R - 304)}{4 \times 578} = \frac{Y_{1R} - Y_{1\infty}}{1 - Y_{1R}} \approx \frac{Y_{1R} - 0.02}{0.98}$$

If  $T_\infty \approx 324K$  as an average through the compressor,

$$\frac{\Delta T}{h_v/c_p} = \frac{20 - \delta t}{2312} = \frac{Y_{1R} - Y_{1\infty}}{1 - Y_{1R}} = \frac{0.021 e^{26.4 \frac{\delta t}{304}} - 0.02}{0.98}$$

If we guess  $\delta t = 4^\circ C$

$$\frac{16}{2312} = 0.0069, \text{ and } \frac{0.021 (1.35) - 0.02}{0.98} = 0.0082$$

which is close enough, so  $\delta t = 4^\circ C$ , and  $Y_{1R} \approx 0.03$ . Note, if the compressor exit temperature were used for  $T_\infty$ , the  $\Delta T$  would be larger, and also  $\delta t$ . Also, if the inlet humidity was reduced, the  $\Delta T$  would be about the same, but  $\delta t$  would be somewhat smaller in order to make up for the loss of  $T_{1\infty}$ .

In any case, the evaporation rate is computed from

$$\frac{\Delta T}{h_v/c_p} = e^{\dot{m}/4\pi\bar{\rho}\alpha R} - 1 = e^{v_R R/\alpha} - 1$$

Therefore if  $\Delta T/h_v/c_p \ll 1$ ,  $\Delta T/h_v/c_p \approx \dot{m}/4\pi\bar{\rho}\alpha R$

$$\dot{m} = -\rho_L \frac{d\pi D^3/6}{dt} = \frac{\Delta T}{h_v/c_p} 4\pi\bar{\rho}\alpha R$$

and

$$\frac{dD^2}{dt} = 8 \frac{\bar{\rho}}{\rho_L} \alpha \frac{\Delta T}{h_v/c_p}$$

so that the time required to evaporate a drop size of  $D_o$  is,

$$t_o \approx \frac{\rho_L}{\bar{\rho}} \frac{D_o^2}{8\alpha} \left( \frac{h_v/c_p}{\Delta T} \right)$$

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Thus if  $\alpha = 5 \times 10^5 \text{ m}^2/\text{s}$  and  $D_o = 10^{-5} \text{ m}$ , i.e. 10 microns.

$$t_o \approx \frac{1.7 \times 10^4}{5 \times 10} \cdot 10^{-10} = 34 \text{ ms}$$

From this baseline we may consider two changes, lower inlet humidity and behavior at the higher pressure stages of the compressor. If the inlet humidity was zero, the  $\delta t$  will be negative at about -6K, so that

$$\frac{24 - \delta t}{2312} = 0.013 \text{ and } 0.021 \times 0.47 \approx 0.010$$

If  $\delta t = -4$

$$\frac{28}{2312} = 0.012 \text{ and } 0.021 \times 0.65 \approx 0.014$$

So, a solution is,

$$\delta t \approx -5 \text{ \& } \Delta T/h_v/c_p \approx 0.0125$$

and

$$\frac{dD^2}{dt} = 8 \frac{\bar{\rho}}{\rho_L} \alpha \left( \frac{\Delta T}{h_v/c_p} \right)$$

So that

$$t_o = \frac{\rho_L}{\bar{\rho}} \frac{D_o^2}{8\alpha} \frac{(h_v/c_p)}{\Delta T} \approx 20 \text{ ms}$$

Therefore zero inlet humidity will act to increase the evaporation rate by 40%.

In the later stages of the compressor, where the pressure ratio might be 10, we have

$$\frac{T_2}{T_o} \approx (10)^{0.286} \approx 2.0 \quad T_2 = 622 \text{ K}$$

and because of the inlet humidity,  $P_v = 4.75 \text{ psi}$ ,  $T_{\text{sat}} = 160^\circ\text{F}$  and  $h_v = 478 \text{ cal/g}$

Therefore, referring the temperature to 344K, (160°F)

$$p_1(\delta t) = 4.75 e^{19(\delta t/344)} \quad \& \quad Y_{IR}(\delta t) = 0.020 e^{19\left(\frac{\delta t}{344}\right)}$$

$$\frac{(T_{\infty} - 344) - (T_R - 344)}{4 \times 478} = \frac{Y_{IR} - Y_{\infty}}{1 - Y_{IR}} = \frac{Y_{IR} - 0.02}{0.98}$$

Again, finding the solution to the surface temperature, surface vapor pressure relationship,

$$\frac{278 - \delta t}{1912} \approx 0.02 (0.055 \delta t) / 0.98$$

But, if  $\delta t$  is more than 10°C, the small exponent approximation is not good, and a more exact iteration is needed. Doing this, at

$$\delta t = 35^\circ\text{C} \quad 243 \approx 38.2 (7.5 - 1) = 248$$

Hence  $\delta t \approx 35^\circ\text{C}$  is a solution that matches  $T_R$  with  $Y_{IR}$  at the droplet surface, and  $\Delta T/h_v/c_p = 0.127$ .

Therefore, for the high pressure stages of the compressor;

$$t_o \approx \frac{\bar{p}_L}{\bar{p}} \frac{D_o^2}{8\alpha} \frac{(h_v/c_p)}{\Delta T} \approx 2 \text{ ms}$$

So, the drop will evaporate 10 times faster in the higher pressure and higher temperature regions of the compressor. It might be noted that the inlet humidity will not have a great effect on this result. Also a droplet temperature of 380°K = 225°F, the vapor pressure on the surface is 19 psia, so  $Y_{IR} = 0.08$  from the steam tables, and  $Y_{IR} \approx 1.6 \times 0.08$  from the above Clausius Clapeyron approximation. Thus, if steam tables were used for the iterative solution, a somewhat higher  $\delta t$  would result. This will not have a large effect on the conclusions regarding the evaporation rate.

#### IV. Closing Observations

For evaporative cooling of the air flow through a gas turbine compressor by water fog injection, the following comments may be made:

1. A set of analytics is available to compute an approximation to the evaporation cooling process. Solutions depend on a number of assumptions concerning simple transport properties, and quasi-steady behavior, but are educational first approximations to the true behavior.

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2. The solutions show the evaporation rates to be very droplet size and temperature sensitive. For standard gas turbine inlet conditions, and 10 micron diameter drops at the inlet, it appears that, for an approximate particle total through transport time of 10 ms, the cooling effect will be delayed to the later stages of compression.
  3. For a 100°F day with 50% humidity, the mass fraction of water in the inlet air is 0.02, which corresponds to a total water mass flow rate about equal to the full power fuel flow rate. Adding an equal amount of water to the inlet as a fog would provide a cooling capacity of 40°K. From an engine power output consideration, it would be most beneficial to achieve this cooling as early in the compression step as possible. But, evaporative cooling anywhere should be attractive, the only disadvantage being the large amounts of pure water required.
  4. Water drops larger than 10 microns may pass through the compressor and into the combustor, eventually giving additional work for additional fuel. Cycle analysis will be needed to clarify the rematched engine changes due to cooling, and to liquid carry over into the combustor.
  5. With the large amounts of water addition, existing data suggests that NO<sub>x</sub> reductions will be quite significant, as already demonstrated by the WLN units currently used by land-based utilities.
  6. It seems reasonable to explore further the thermodynamic path of droplet evaporation, and gas cooling, in an axial flow compressor. Such parameters as transfer coefficients, droplet size distribution, blade angle variations between wet and dry, gas temperatures, etc. need to be understood.

In summary, the concept of water fog evaporative cooling of the inlet air for a ship gas turbine appears feasible and attractive from a theoretical standpoint. The definition of a practical set of shipboard equipment and operating techniques using this concept should be the next step for assessment by the Navy.

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